3rd Grade Common Core State Standards

Flip Book

This document is intended to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "Flip Book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of mathematics.

Construction directions:

Print single-sided on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

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1. **Make sense of problems and persevere in solving them.**
In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2. **Reason abstractly and quantitatively.**
Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
3. **Construct viable arguments and critique the reasoning of others.**
In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. **Model with mathematics.**
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. **Use appropriate tools strategically.**
Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
6. **Attend to precision.**
As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.
7. **Look for and make use of structure. (Deductive Reasoning)**
In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).
8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _______ related to ________? What is the relationship between ______and ______? What does_______mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a ______ show us that _____may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?

Critical Areas for Mathematics in 3rd Grade

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $^1/_2$ of the paint in a small bucket could be less paint than $^1/_3$ of the paint in a larger bucket, but $^1/_3$ of a ribbon is longer than $^1/_5$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Represent and solve problems involving multiplication and division.

Standard: **3.OA.1**. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.

Connections: **(3.OA.1-4)**

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplication and division and strategies for multiplication and division within 100**.

Connect this domain with understanding properties of multiplication and the relationship between multiplication and division. (Grade 3 OA 5 – 6)

The use of a symbol for an unknown is foundational for letter variables in Grade 4 when representing problems using equations with a letter standing for the unknown quantity (Grade 4 OA 2 and OA 3).

Explanations and Examples

This standard interpret products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Multiplication is seen as "groups of" and problems such as 5 x 7 refer to 5 groups of 7. However, it is important for teachers to understand there are several ways in which we can think of multiplication: 1)Multiplication is often thought of as repeated addition of equal groups. While this definition works for some sets of numbers, it is not particularly intuitive or meaningful when we think of multiplying 3 by 1/2, for example, or 5 by -2. In such cases, it may be helpful to widen the idea of grouping to include evaluation of part of a group. This concept is related to partitioning (which, in turn, is related to division). Ex: three groups of five students can be read as 3 • 5, or 15 students, while half a group of 10 stars can be represented as 1/2 • 10, or 5 stars. These are examples of partitioning; each one of the three groups of five is part of the group of 15, and the group of 5 stars is part of the group of 10.
2)A second concept of multiplication is that of rate or price. Ex: If a car travels four hours at 50 miles per hour, then it travels a total of 4 • 50, or 200 miles; if CDs cost eight dollars each, then three CDs will cost 3 • $8, or $24. 3)A third concept of multiplication is that of multiplicative comparison. Ex: Sara has four CDs, Joanne has three times as many as Sara, and Sylvia has half as many as Sara. Thus, Joanne has 3 • 4, or 12 CDs, and Sylvia has 1/2 • 4, or 2 CDs.

Example for this level (3.OA.1):

Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15.
Describe another situation where there would be 5 groups of 3 or 5 x 3.

Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

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To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5 x 6) students interpret the expression using a multiplication context. (See Table 2, page 58) They should begin to use the terms, *factor* and *product,* as they describe multiplication.

Instructional Strategies (3.OA.1-4)

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This strategy is a foundation for multiplication because students should make a connection between repeated addition and multiplication.

Students need to experience problem-solving involving equal groups (whole unknown or size of group is unknown) and multiplicative comparison (unknown product, group size unknown or number of groups unknown) as shown in Table 2 of the Common Core State Standards for Mathematics, page 74.

Encourage students to solve these problems in different ways to show the same idea and be able to explain their thinking verbally and in written expression. Allowing students to present several different strategies provides the opportunity for them to compare strategies.

Sets of counters, number lines to skip count and relate to multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. Students should represent the model used as a drawing or equation to find the solution.

Show a variety of models of multiplication. (i.e. 3 groups of 5 counters can be written as 3 × 5. Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.

Have students create multiplication problem situations in which they interpret the product of whole numbers as the total number of objects in a group and write as an expression. Also, have students create division-problem situations in which they interpret the quotient of whole numbers as the number of shares.

Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷ ☐ = 3, students should use knowledge of the related multiplication fact of 3 × 9 = 27. They should ask themselves questions such as, "How many 3s are in 27 ?" or "3 times what number is 27?" Have them justify their thinking with models or drawings.

Instructional Resources/Tools

Sets of counters
Number lines to skip count and relate to multiplication
Arrays Table 2. Common multiplication and division situations (Common Core State Standards for Mathematics 2010) (page 58 in this document).
National Council of Teachers of Mathematics, Illuminations: Exploring equal sets.
This four-part lesson encourages students to explore models for multiplication, the inverse of multiplication, and representing multiplication facts in equation form.
National Council of Teachers of Mathematics, Illuminations: All About Multiplication
In this four-lesson unit, students explore several meanings and representation of multiplications and learn about properties of operations for multiplication.

Common Misconceptions: (3.OA.1-4)

Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = ☐ x 3.

Students also think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations. The use of models is essential in helping students eliminate this understanding.

The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation. Presenting students with multiple situations in which they select the symbol and explain what it represents will counter this misconception.

3.OA.1

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Represent and solve problems involving multiplication and division.

Standard: **3.OA.2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.

Connections:

See 3.OA.1

Explanations and Examples:

This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.

Partition models focus on the question, "How many in each group?" A context for partition models would be:
There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?

Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context or measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?

Students need to recognize the operation of division in two different types of situations. One situation requires determining how many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor).

To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division. (See Table 2 page 74)

Students may use interactive whiteboards to create digital models.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Represent and solve problems involving multiplication and division.

Standard: **3.OA.3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 2.)

Standards for Mathematical Practices to be emphasized:

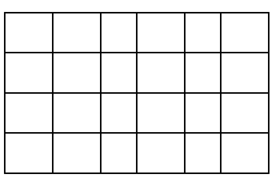
MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.

Connections:
See 3.OA.1

Explanations and Examples:

This standard references various strategies that can be used to solve word problems involving multiplication & division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many brownies does each person receive? *(4 x 9 = 36, 36 ÷ 6 = 6).*
Table 2 , page 58 of this document, gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.

Examples of multiplication:
There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there? This task can be solved by drawing an array by putting 6 desks in each row. This is an array model



This task can also be solved by drawing pictures of equal groups. 4 groups of 6 equals 24 objects

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A student could also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24.   Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom."

A number line could also be used to show equal jumps.

Students in third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in third grade.

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Examples of Division:
There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class *(÷ 4 = 6. There are 24 students in the class).*

Determining the number of objects in each share (partitive division, where the size of the groups is unknown):

Example:
The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?
Determining the number of shares (measurement division, where the number of groups is unknown)

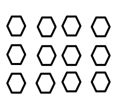
Example:
Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?
Starting Day 1 Day 2 Day 3 Day 4 Day 5 Day 6
24 24 – 4 = 20 20 – 4 = 16 16 – 4 = 12 12 – 4 = 8 8 – 4 = 4 4 – 4 = 0

Solution: The bananas will last for 6 days.

Students use a variety of representations for creating and solving one-step word problems, i.e., numbers, words, pictures, physical objects, or equations. They use multiplication and division of whole numbers up to 10 x10. Students explain their thinking, show their work by using at least one representation, and verify that their answer is reasonable.

Word problems may be represented in multiple ways:

- Equations: 3 x 4 = ?**,** 4 x 3 = ?, 12 ÷ 4 = ? and 12 ÷ 3 = ?
- Array:



- Equal groups

- Repeated addition: 4 + 4 + 4 or repeated subtraction
- Three equal jumps forward from 0 on the number line to 12 or three equal jumps backwards from 12 to 0

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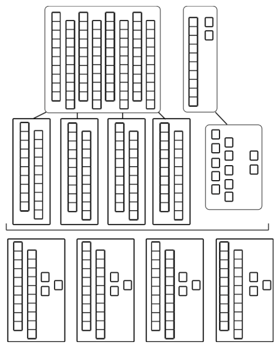
Examples of division problems:

- Determining the number of objects in each share (partitive division, where the size of the groups is unknown):
  - The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



- Determining the number of shares (measurement division, where the number of groups is unknown)
  - Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**
24	24-4=20	20-4=16	16-4=12	12-4=8	8-4=4	4-4=0

Solution: The bananas will last for 6 days.

Students may use interactive whiteboards to show work and justify their thinking.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Represent and solve problems involving multiplication and division.

Standard**: 3.OA.4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections:
See 2.OA.1

Explanations and Examples:

This standard refers to Table 2 page 58 of this document and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product (3 x 6 = ? or 18 ÷ 3 = 6). The more difficult problem structures include Group Size Unknown (3 x ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? x 6 = 18, 18 ÷ 6 = 3). The focus of 3.OA.4 goes beyond the traditional notion of *fact families*, by having students explore the **inverse relationship** of multiplication and division.

Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Example:
Solve the equations below:
24 = ? x 6
72 ÷ □ = 9

Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*

This standard is strongly connected to 3.AO.3 when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.

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Students apply their understanding of the meaning of the equal sign as ”the same as” to interpret an equation with an unknown. When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Examples:

- Solve the equations below:
  24 = ? x 6
  $72 \div \Delta = 9$
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*

Students may use interactive whiteboards to create digital models to explain and justify their thinking.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

Standard: **3.OA.5.** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections (3.OA.*5-6*)

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplication and division and strategies for multiplication and division within 100**.

Explanations and Examples:

This standard references properties of multiplication. While students DO NOT need to use the formal terms of these properties, student should understand that properties are rules about how numbers work, students do need to be flexibley and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70.

The commutative property (order property) states that the order of numbers does not matter when adding or multiplying numbers. For example, if a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20.
The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. **There is no "fixed" way to write the dimensions of an array as rows x columns or columns x rows.**

Students should have flexibility in being able to describe both dimensions of an array.

X X X X		X X X X X
X X X X	4x5	X X X X X
X X X X	or	X X X X X
X X X X	5x4	X X X X X
X X X X		

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Students should be introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product.

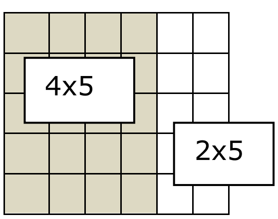
Here are ways that students could use the distributive property to determine the product of 7 x 6. Again, students should use the distributive property, but can refer to this in informal language such as "breaking numbers apart".

<u>Student 1</u>
7 x 6
7 x 5 = 35
7 x 1 = 7
35 + 7 = 42

<u>Student 2</u>
7 x 6
7 x 3 = 21
7 x 3 = 21
21 + 21 = 42

<u>Student 3</u>
7 x 6
5 x 6 = 30
2 x 6 = 12
30 + 12 = 42

Another example of the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem 6 x 5= ?, students can decompose the 6 into a 4 and 2, and reach the answer by multiplying 4 x 5 = 20 and 2 x 5 =10 and adding the two products (20+10=30).



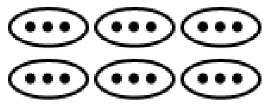
To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.
- 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)
- 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)
- 3 x 6 = 6 x 3 (Commutative Property)
- 8 ÷ 2 ≠ 2 ÷ 8 (Students are only to determine that these are not equal)
- 2 x 3 x 5 = 6 x 5
- 10 x 2 < 5 x 2 x 2
- 2 x 3 x 5 = 10 x 3
- 1 x 6 > 3 x 0 x 2

Continued next page

Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1, never by 0. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

Models help build understanding of the commutative property:

Example: 3 x 6 = 6 x 3
In the following diagram it may not be obvious that 3 groups of 6 is the same as 6 groups of 3. A student may need to count to verify this.

 is the same quantity as 

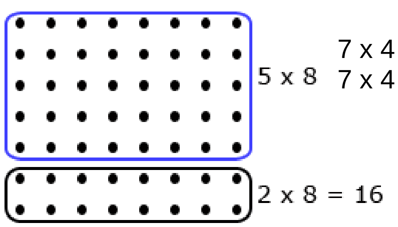
Different representation:
An array explicitly demonstrates the concept of the commutative property.

4 rows of 3 or 4 x 3

3 rows of 4 or 3 x 4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. Students should learn that they can decompose either of the factors. It is important to note that the students may record their thinking in different ways.

5 x 8 = 40
2 x 8 = 16
56



7 x 4 = 28
7 x 4 = 28
56

Continued next page

Instructional Strategies (3.OA.5-6)
Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 (15) is the same as the result of multiplying 5 x 3 (15).

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. (See example above where students split an array into smaller arrays and add the sums of the groups.

Students' understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

Instructional Resources/Tools
National Council of Teachers of Mathematics, Illuminations:– Multiplication: It's in the Cards - Students skip-count and examine multiplication patterns. They also explore the commutative property of multiplication.

Nctm.org (Illuminations):- Multiplication: It's in the Cards: Looking for Calculator Patterns - Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Understand properties of multiplication and the relationship between multiplication and division.

Standard: **3.OA.6.** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.7. Look for and make use of structure.

Connections:

See 3.OA.5

Explanations and Examples:

This standard refers the Glossary on page 58, Table 2 and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor in multiplication problems.

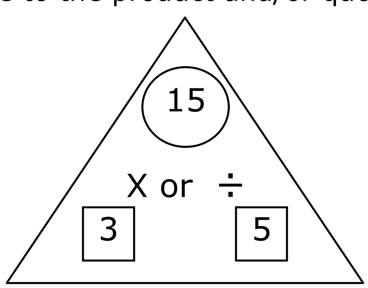
Example:
A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Example:
- 3 x 5 = 15   5 x 3 = 15
- 15 ÷ 3 = 5   15 ÷ 5 = 3

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:
- 3 x 5 = 15   5 x 3 = 15
- 15 ÷ 3 = 5   15 ÷ 5 = 3



Students use their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 32 ÷ □ = 4, students may think:
- 4 groups of some number is the same as 32
- 4 times some number is the same as 32
- I know that 4 groups of 8 is 32 so the unknown number is 8
- The missing factor is 8 because 4 times 8 is 32.

Equations in the form of a ÷ b = c and c = a ÷ b need to be used interchangeably, with the unknown in different positions.

3.OA.6

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Multiply and divide within 100

Standard: **3.OA.7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Standards for Mathematical Practices to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplication and division and strategies for multiplication and division within 100**.

Explanations and Examples:

This standard uses the word fluently, which means accuracy, efficiency (**using a reasonable amount of steps and time**), and flexibility (using strategies such as the distributive property). "Know from memory" does **not** mean focusing only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9).

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

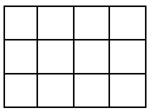
- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of __ and knowing how many groups have been counted)
- Square numbers (ex: 3 x 3)
- Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
- Missing factors

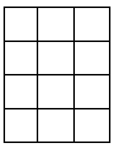
General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms. *(Problems presented horizontally encourage solving mentally).*

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

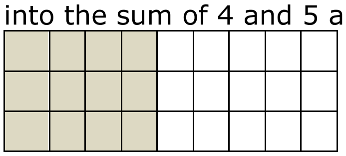
Continued next page

Instructional Strategies:
Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.
In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. **This is called unitizing**, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.
As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.





To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).



The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division.

Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

Instructional Resources/Tools
Unifix cubes or cubes
Grid or graph paper
Sets of counters

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Standard: **3.OA.8**. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.

Connections:

This cluster is connected to the Third Grade Critical Area of Focus #1, **Developing understanding of multiplication and division and strategies for multiplication and division within 100**.
Represent and solve problems involving multiplication and division. (Grade 3 OA 1 – 4)
Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 1 -3)

Explanations and Examples:

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities.
Example:
Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or
100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example:
Here are some typical estimation strategies for the problem:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

Student 1
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2
I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with
500.

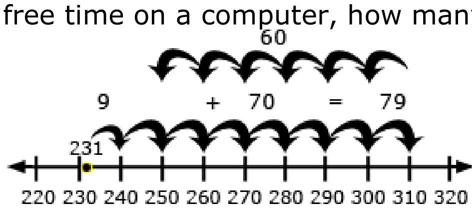
Student 3 continued next page

Student 3
I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students should be expected to explain their thinking in arriving at the answer.

It is important that students be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use.

Examples:
- Jerry earned 231 points at school last week. This week he earned 79 points. If he uses 60 points to earn free time on a computer, how many points will he have left?



A student may use the number line above to describe his/her thinking, "231 + 9 = 240 so now I need to add 70 more. 240, 250 (10 more), 260 (20 more), 270, 280, 290, 300, 310 (70 more). Now I need to count back 60. 310, 300 (back 10), 290 (back 20), 280, 270, 260, 250 (back 60)."

A student writes the equation, 231 + 79 – 60 = m and uses rounding (230 + 80 – 60) to estimate.

A student writes the equation, 231 + 79 – 60 = m and calculates 79-60 = 19 and then calculates 231 + 19 = m.

- The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Write an equation representing the cost of the field trip and determine the price of one wristband.

w	w	13
63		

The above diagram helps the student write the equation, w + w + 13 = 63. Using the diagram, a student might think, "I know that the two wristbands cost $50 ($63-$13) so one wristband costs $25." To check for reasonableness, a student might use front end estimation and say 60-10 = 50 and 50 ÷ 2 = 25.

When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Estimation strategies include, but are not limited to:
- using benchmark numbers that are easy to compute
- front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts)

rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values)

Instructional Strategies for 3.OA.8-9 next page

Instructional Strategies: (3.OA.8-9)
Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems. Problems should be presented on a regular basis as students work with numbers and computations.

Researchers and mathematics educators advise against providing "key words" for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it. (M.Burns)

Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies.

Students can use base–ten blocks on centimeter grid paper to construct rectangular arrays to represent problems.

Students are to identify arithmetic patterns and explain them using properties of operations. They can explore patterns by determining likenesses, differences and changes. Use patterns in addition and multiplication tables.

Instructional Resources/Tools
Nctm.org (Illuminations): Times. Students can also look for patterns in the table.
National Council of Teachers of Mathematics, Illuminations:– Multiplication: It's in the Cards - Students skip-count and examine multiplication patterns. They also explore the commutative property of multiplication.

Nctm.org (Illuminations):– Multiplication: It's in the Cards: Looking for Calculator Patterns - Students use a web-based calculator to create and compare counting patterns using the constant function feature of the calculator. Making connections between multiple representations of counting patterns reinforces students understanding of this important idea and helps them recall these patterns as multiplication facts. From a chart, students notice that multiplication is commutative.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Standard: **3.OA.9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections:

See 3.OA.8

Explanations and Examples:

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations.
Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).
- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

Using a multiplication table, highlight a row of numbers and ask students what they notice about the highlighted numbers.
Explain a pattern using properties of operations.
When (commutative property) one changes the order of the factors they will still gets the same product, example 6 x 5 = 30 and 5 x 6 = 30.
Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?
Student: The product will always be an even number.
Teacher: Why?

In an addition table ask what patterns they notice.? Explain why the pattern works this way?

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.
Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

Addend	**Addend**	**Sum**
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
?	?	?
?	?	?
?	?	?
20	0	20

Third Grade Operations and Algebraic Thinking
Extended Common Core State Standards Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.

North Carolina DOE

Third Grade Mathematics
Operations and Algebraic Thinking (3.OA)

	Common Core State Standards	Essence		Extended Common Core
	Represent and solve problems involving multiplication and division	**Represent and solve problems**		**Represent and solve problems**
Cluster	1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* 2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 ×? = 48, 5 = ÷ 3, 6 × 6 =?*		Cluster	1. Compose and decompose numbers on both sides of the equal sign to show equality. 2. Solve addition and subtraction problems when result is unknown (i.e. 8 + 2 = , 6 – 3 =).

Understand properties of multiplication and the relationship between multiplication and division		**Build foundation for multiplication through repeated addition**	**Represent repeated addition**	
	5. Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*			3. Build models that represent repeated addition. (i.e., 2 groups of 4 is the same quantity as 4 + 4) 4. Share equally collections of up to 30 items between 2 to 4 people to solve real life story problems.

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

Standard: **3.NBT.1.** Use place value understanding to round whole numbers to the nearest 10 or 100.

Standards for Mathematical Practices to be emphasized:

MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(3.NBT.1-3)**

This cluster is connected to the Third Grade Critical Area of Focus #1**, Developing understanding of multiplication and division and strategies for multiplication and division within 100**. Additionally, the content in this cluster goes beyond the critical areas to address solving multi-step problems.

The rounding strategies developed in third grade will be expanded in grade four with larger numbers. Additionally, students will formalize the rules for rounding numbers with the expansion of numbers in fourth grade.

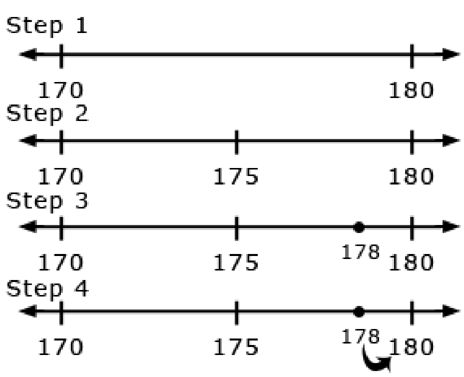
In fourth grade the place value concepts developed in grades K-3 will be expanded to include decimal notation. Understand place value. (Grade 2 NBT 1 – 4 and Grade 2 NBT 5 – 9)

Explanations and Examples:

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding.
The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Students learn when and **why** to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, at this level the number is rounded up.

Example: Round 178 to the nearest 10.



Step 1: The answer is either 170 or 180.

Step 2: The halfway point is 175.

Step 3: 178 is between 175 and 180.

Step 4: Therefore, the rounded number is 180.

Instructional Strategies next page

Instructional Strategies

Prior to implementing rules for rounding students need to have opportunities to investigate place value. A strong understanding of place value is essential for the developed number sense and the subsequent work that involves rounding numbers.

Building on previous understandings of the place value of digits in multi-digit numbers, place value is used to round whole numbers. Dependence on learning rules can be eliminated with strategies such as the use of a number line to determine which multiple of 10 or of100, a number is nearest (5 or more rounds up, less than 5 rounds down). As students' understanding of place value increases, the strategies for rounding are valuable for estimating, justifying and predicting the reasonableness of solutions in problem-solving.

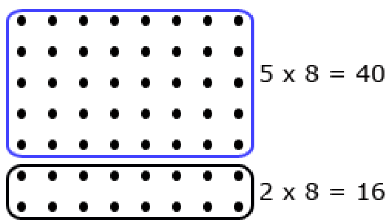
Strategies used to add and subtract two-digit numbers are now applied to fluently add and subtract whole numbers within 1000. These strategies should be discussed so that students can make comparisons and move toward efficient methods.

Number sense and computational understanding is built on a firm understanding of place value.

Understanding what each number in a multiplication expression represents is important. Multiplication problems need to be modeled with pictures, diagrams or concrete materials to help students understand what the factors and products represent. The effect of multiplying numbers needs to be examined and understood.

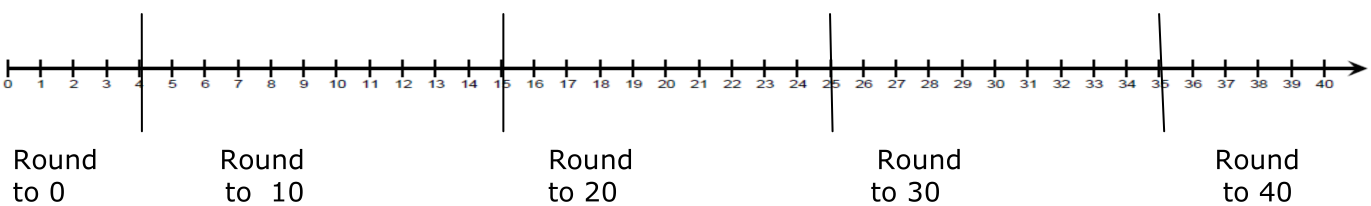
The use of area models is important in understanding the properties of operations of multiplication and the relationship of the factors and its product. Composing and decomposing area models is useful in the development and understanding of the distributive property in multiplication.

$$\begin{array}{r} 40 \\ \underline{+16} \\ 56 \end{array}$$



Continue to use manipulative like hundreds charts and place-value charts. Have students use a number line or a roller coaster example to block off the numbers in different colors.

For example this chart show what numbers will round to the tens place.



Common Misconceptions: (3.NBT.1-3)

The use of terms like "round up" and "round down" confuses many students. For example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

3.NBT.1					

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

Standard: **3.NBT.2.** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standards for Mathematical Practices to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

See 3.NBT.1

Explanations and Examples:

This standard refers to fluently, which means accuracy, efficiency (using a reasonable number of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard/traditional algorithm. Third grade students should have experiences beyond the standard/traditional algorithm.

Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.

Example:
There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1
100 + 200 = 300
70 + 20 = 90
8 + 5 = 13
300 + 90 + 13 =
403 students

Student 2
I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.

Student 3
I know the 75 plus 25 equals 100. I then added
1 hundred from 178 and
2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

Student 4
178+225=?
178+200=378
378+20+398
398+5=403

Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. An interactive whiteboard or document camera may be used to show and share student thinking.

Example:

- Mary read 573 pages during her summer reading challenge. She was only required to read 399 pages. How many extra pages did Mary read beyond the challenge requirements?

Students may use several approaches to solve the problem including the traditional algorithm. Examples of other methods students may use are listed below:

- 399 + 1 = 400, 400 + 100 = 500, 500 + 73 = 573, therefore 1+ 100 + 73 = 174 pages (Adding up strategy)
- 400 + 100 is 500; 500 + 73 is 573; 100 + 73 is 173 plus 1 (for 399, to 400) is 174 (Compensating strategy)
- Take away 73 from 573 to get to 500, take away 100 to get to 400, and take away 1 to get to 399. Then 73 +100 + 1 = 174 (Subtracting to count down strategy)
- 399 + 1 is 400, 500 (that's 100 more). 510, 520, 530, 540, 550, 560, 570, (that's 70 more), 571, 572, 573 (that's 3 more) so the total is 1 + 100 + 70 + 3 = 174 (Adding by tens or hundreds strategy)

Domain: **Number and Operations in Base Ten (NBT)**
Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)
Standard: **3.NBT.3.** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
Standards for Mathematical Practices to be emphasized: MP.2. Reason abstractly and quantitatively. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Connections: See 3.NBT.1
Explanations and Examples: This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200. Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10-90. They apply their understanding of multiplication and the meaning of the multiples of 10. For example, 30 is 3 tens and 70 is 7 tens. They can interpret 2 x 40 as 2 groups of 4 tens or 8 groups of ten. They understand that 5 x 60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding they begin to recognize the patterns in multiplying by multiples of 10. Students may use manipulatives, drawings, document camera, or interactive whiteboard to demonstrate their understanding.

Third Grade Number and Operations in Base Ten
Extended Common Core State Standards Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.

North Carolina DOE

Third Grade Mathematics Operations in Base Ten (3.NBT)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Use place value understanding and properties of operations to perform multi-digit arithmetic		**Understand place value**	**Use place value understanding to add and subtract**	
Cluster	1. Use place value understanding to round whole numbers to the nearest 10 or 100. 2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.		Cluster	1. Use a number line (0-30) to determine the number 1 more and 1 less and 2 more and 2 less. 2. Illustrate ten and some more with numbers 11-30 using objects (bundles of ten). 3. Use part-part-whole relationships (including 2 or more parts), to compose and decompose numbers (0-30). 4. Compare numbers (0-30) in relationship to benchmark numbers 5 and 10. 5. Compare sets of objects (0-30) by their relative magnitude (e.g., more, less, equal, one more, one less, bigger, smaller). 6. Use estimation to determine if a set of objects is "more than 10," "less than 10," or "about the same as 10." 7. Use language and symbols (subtract, add, equal) to describe addition and subtraction problems. 8. Use addition and subtraction symbols in solving problems up to 30.

Domain: **Number and Operations—Fractions (NF)** (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Cluster: Develop understanding of fractions as numbers.

Standard: **3.NF.1.** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics
MP.7. Look for and make use of structure.

Connections: **(3.NF.1-3)**

This cluster is connected to the Third Grade Critical Area of Focus #2, Developing understanding of fractions, especially unit fractions(fractions with numerator 1).

Partitioning traditional shapes into equal parts. (Grade 1 G 3)

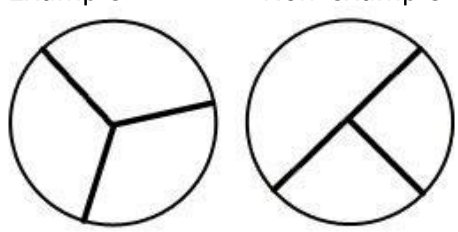
Explanations and Examples:

This standard refers to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares) and number lines. **Set models** (parts of a group) are not explored in Third Grade. In 3.NF.1 students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1. For example, the fraction 3/5 is composed of 3 pieces that each have a size of 1/5.

Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized

Example Non-example



- The number of equal parts tell how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.

The size of the fractional part is relative to the whole.

- The number of children in one-half of a classroom is different than the number of children in one-half of a school. (the whole in each set is different therefore the half in each set will be different)

- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
    - ¾ means that there are 3 one-fourths.
    - Students can count *one fourth, two fourths, three fourths.*

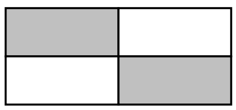
Students express fractions as "fair sharing", parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.

Continued next page

To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.

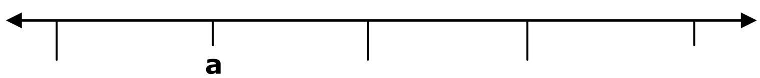
Examples: (Area or Region Model)

- Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
- Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?
- What fraction of the rectangle is shaded? How might you draw the rectangle in another way but with the same fraction shaded?



Solution: $\frac{2}{4}$ or $\frac{1}{2}$

What fraction does the letter **a** represent? (Linear Model) Explain your thinking.

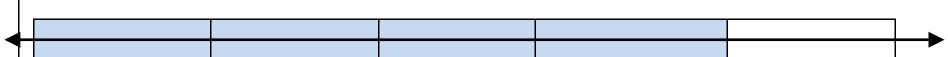


<u>Instructional Strategies</u> (3.NF.1-3)

This is the initial experience students will have with fractions and is best done over time. Students need many opportunities to discuss fractional parts using concrete models to develop familiarity and understanding of fractions. Expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Understanding that a fraction is a quantity formed by part of a whole is essential to number sense with fractions. Fractional parts are the building blocks for all fraction concepts. Students need to relate dividing a shape into equal parts and representing this relationship on a number line, where the equal parts are between two whole numbers. Help students plot fractions on a number line, by using the meaning of the fraction. For example, to plot 4/5 on a number line, there are 5 equal parts with 4 copies of the 5 equal parts.

5 equal parts make the whole



0 1/5 2/5 3/5 4/5 5/5 or 1

4 copies of the 5 equal parts represent the fractional amount

As students counted with whole numbers, they should also count with fractions. Counting equal-sized parts helps students determine the number of parts it takes to make a whole and recognize fractions that are equivalent to whole numbers.

Continued next page

Common Misconceptions: (3.NF.1)

The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is. The use of different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.

Students think all shapes can be divided the same way. Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:

1. Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
2. Fold in this manner two more times.
3. Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

Domain: **Number and Operations—Fractions (NF)** (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Cluster: Develop understanding of fractions as numbers.

Standard: **3.NF.2.** Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

Standards for Mathematical Practices to be emphasized:

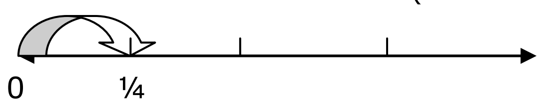
MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics
MP.7. Look for and make use of structure.

Connections:

See 3.NF.1

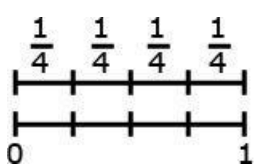
Explanations and Examples:

The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that ½ is between 0 and 1).
In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (**3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (**3.NF.2b**).

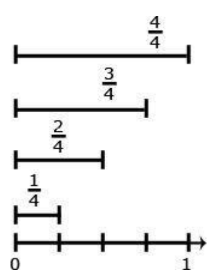


Students transfer their understanding of parts of a whole to partition a number line into equal parts. There are two new concepts addressed in this standard which students should have time to develop.

1. On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.



2. Students label each fractional part based on how far it is from zero to the endpoint.



An interactive whiteboard may be used to help students develop these concepts.

Domain: **Number and Operations—Fractions (NF)** (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)

Cluster: Develop understanding of fractions as numbers.

Standard: **3.NF.3.** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

See 3.NF.1

Explanations and Examples:

An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than ½ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

3.NF.3a and **3.NF.3b** These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of $\frac{a}{1}$

Example:

If 6 brownies are shared between 2 people, how many brownies would each person get?

This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than ¼ of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

Continued next page

In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, ½ of a large pizza is a different amount than ½ of a small pizza. Students should be given opportunities to discuss and reason about which ½ is larger.

An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction with the larger numerator has the larger number of equal parts.

$$\frac{2}{6} < \frac{5}{6}$$

To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces.

$$\frac{3}{8} < \frac{3}{4}$$

3.NF.3

Third Grade Mathematics
Extended Common Core State Standards Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication. North Carolina DOE

Third Grade Mathematics Number and Operations-Fractions (3.NF)			
Common Core State Standards		**Essence**	**Extended Common Core**
Develop understanding of fractions as numbers		Understand fractions	Develop understanding of simple fractions
Cluster	**1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.** **2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.** **a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.** **b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.** **3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.** **a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.** **b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.** **c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.** **d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.**		Cluster 1. Identify whole and half using concrete models (use continuous and discrete items). 2. Use symbolic representation for each equal part.

EXTENDED 3.NF

Domain: **Measurement and Data (MD)**

Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Standard: **3.MD.1.** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.5. Uses appropriate tools strategically.
MP.6. Attend to precision.

Connections: **(3.MD.1-2)**

This cluster extends the Third Grade Critical Areas of Focus, **Solving multi-step problems**.

Work with time and money. Grade 2 MD 7

Explanations and Examples:

This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).

Students in second grade learned to tell time to the nearest five minutes. In third grade, they extend telling time and measure elapsed time both in and out of context using clocks and number lines.

Students may use an interactive whiteboard to demonstrate understanding and justify their thinking.

Instructional Strategies next page

Instructional Strategies (3.MD.1-2)

A clock is a common instrument for measuring time. Learning to tell time has much to do with learning to read a dial-type instrument rather than with time measurement.

Students have experience in telling and writing time from analog and digital clocks to the hour and half hour in Grade 1 and to the nearest five minutes, using a.m. and p.m. in Grade 2. Now students will tell and write time to the nearest minute and measure time intervals in minutes.

Provide analog clocks that allow students to move the minute hand.

Students need experience representing time from a digital clock to an analog clock and vice versa.

Provide word problems involving addition and subtraction of time intervals in minutes. Have students represent the problem on a number line. Student should relate using the number line with subtraction from Grade 2.

Provide opportunities for students to use appropriate tools to measure and estimate liquid volumes in liters only and masses of objects in grams and kilograms. Students need practice in reading the scales on measuring tools since the markings may not always be in intervals of one. The scales may be marked in intervals of two, five or ten.
Allow students to hold gram and kilogram weights in their hand to use as a benchmark. Use water colored with food coloring so that the water can be seen in a beaker.

Students should estimate volumes and masses before actually finding the measuring. Show students a group containing the same kind of objects. Then, show them one of the objects and tell them its weight. Fill a container with more objects and ask students to estimate the weight of the objects.

Use similar strategies with liquid measures. Be sure that students have opportunities to pour liquids into different size containers to see how much liquid will be in certain whole liters. Show students containers and ask, "How many liters do you think will fill the container?"

If making several estimates, students should make an estimate, then the measurement and continue the process of estimating measure rather than all estimates and then all measures. It is important to provide feedback to students on their estimates by using measurement as a way of gaining feedback on estimates.

Common Misconceptions:

Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams may be read as 81 grams. Students realize it is one away from 80, but do not think of it as 79 grams.

Avoid the use of paper plate clocks. Students need to see the actual relationship between the hour and the minute hand. This is not adequately represented on student made clocks.

Students forget to label the measurement or choose the incorrect unit.

3.MD.1

Domain: **Measurement and Data (MD)**

Cluster: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Standard: **3.MD.2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Table 2).

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively,
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

Connections:

See (3.MD.1)

Explanations and Examples:

This standard asks for students to reason about the units of mass and volume. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Word problems should only be one-step and include the same units.

Example:

Students identify 5 things that have a mass of about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) has a mass of about 100 grams so 10 boxes would have a mass of one kilogram.

Example:

A paper clip has a mass of about a) a gram, b) 10 grams, c) 100 grams?

Foundational understandings to help with measure concepts:
Understand that larger units can be subdivided into equivalent units (partition).
Understand that the same unit can be repeated to determine the measure (iteration).
Understand the relationship between the size of a unit and the number of units needed (compensatory principal).

Students need multiple opportunities "massing" classroom objects and filling containers to help them develop a basic understanding of the size and mass of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter.

Common Misconceptions:

Students often focus on size to determine estimates of mass. They can be confused by a big fluffy object and a tiny dense object. Because students cannot tell actual mass until they have handled an object, it is important that teachers do not ask students to estimate the mass of objects until they have had the opportunity to lift the objects and then make an estimate of the mass.

3.MD.2

Domain: **Measurement and Data (MD)**

Cluster: Represent and interpret data.

Standard: **3.MD.3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.6. Attend to precision.
MP.7. Look for and make use of pattern.

Connections: (3.MD.3-4)

This cluster is connected to the Third Grade Critical Areas of Focus #2, **Developing understanding of fractions, especially unit fractions(fractions with numerator 1)** and goes beyond to address **Solving multi-step problems**.

Represent and solve problems involving multiplication and division. (Grade 3 OA 1 – 4)
Multiply and divide within 100. (Grade 3 OA 7)
Solve problems involving the four operations, and identify and explain patterns in arithmetic. (Grade 3 OA 8 – 9)
Represent and interpret data. (Grade 2 MD 9 – 10)

Explanations and Examples:

Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. Graphs on the next page all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.

While exploring data concepts, students should 1)**P**ose a question, 2)**C**ollect data, 3)**A**nalyze data, and 4)**I**nterpret data ***(PCAI).*** Students should be graphing data that is relevant to their lives

Example:
Pose a question: What are some of the questions that could be asked of the date we see? Students should come up with a question. What is the typical genre read in our class? Collect and organize data: student survey

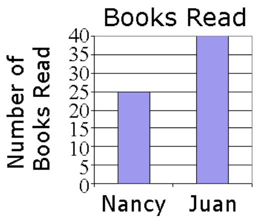
Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, scale, categories, category label, and data. Students need to use both horizontal and vertical bar graphs.

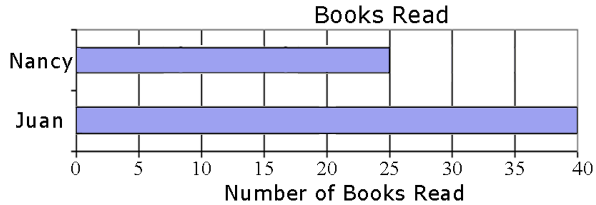
• If you were to purchase a book for the class library which would be the best genre? Why?

Example of Scaled Graph:

Number of Books Read	
Nancy	✧ ✧ ✧ ✧ ✧
Juan	✧ ✧ ✧ ✧ ✧ ✧ ✧ ✧
✧ = 5 Books	

- Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.





Analyze and Interpret data which could include:
- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre?

Instructional Strategies: (3.MD.3-4)
Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, ☐ represents 7 people. If there are three ☐, students should use known facts to determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

Students are to draw picture graphs in which a symbol or picture represents more than one object). Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare quantities and use mathematical concepts and skills. Use symbols on picture graphs that student can easily represent half of, or know how many half of the symbol represents.

Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots. (See line plot example page 44)

Instructional Resources/Tools
Nctm.org (Illuminations): Bar Grapher
This is a NCTM site that contains a bar graph tool to create bar graphs.
Nctm.org (Illuminations): All About Multiplication – Exploring equal sets
Students listen to the counting story, *What Comes in 2's, 3's, & 4's*, and then use counters to set up multiple sets of equal size. They fill in a table listing the number of sets, the number of objects in each set, and the total number in all. They study the table to find examples of the order (commutative) property. Finally, they apply the equal sets model of multiplication by creating pictographs in which each icon represents several data points.
Nctm.org (Illuminations): What's in a Name? – Creating Pictographs. Students create pictographs and answer questions about the data set.

Common Misconceptions: (3.MD.3-4)
Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip- counting when determining the value of a bar since the scale is not in single units.

Domain: **Measurement and Data (MD)**

Cluster: Represent and interpret data.

Standard: **3.MD.4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

<u>Standards for Mathematical Practices to be emphasized:</u>

MP.1. Make sense of problems and persevere in solving them.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP. 6 Attend to precision

<u>Connections</u>:

See 3.MD.3)

<u>Explanations and Examples:</u>

Students in second grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.

This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.

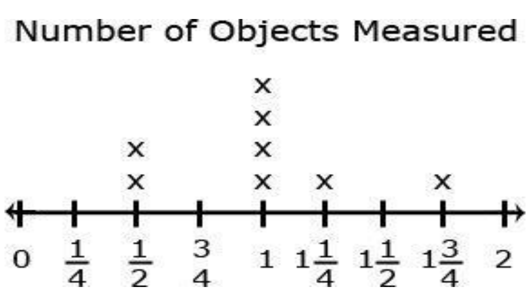
Example:

Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc…

Some important ideas related to measuring with a ruler are:

- The starting point of where one places a ruler to begin measuring
- Measuring is approximate. Items that students measure will not always measure exactly ¼, ½ or one whole inch. Students will need to decide on an appropriate estimate length.
- Making paper rulers and folding to find the half and quarter marks will help students develop a stronger understanding of measuring length

Students generate data by measuring and create a line plot to display their findings. An example of a line plot is shown below:



Domain: **Measurement and Data (MD)**

Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard: **3.MD.5.** Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Standards for Mathematical Practices to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

Connections: **(3.MD.5-7)**

This cluster is connected to the Third Grade Critical Area of Focus #3, **Developing understanding of the structure of rectangular arrays and of area.**

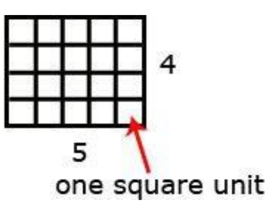
Fluently multiply and divide within 100 (3.OA.2.7).
Distributive property

Explanations and Examples:

These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper.

Students develop understanding of using square units to measure area by:

- Using different sized square units
- Filling in an area with the same sized square units and counting the number of square units
- An interactive whiteboard would allow students to see that square units can be used to cover a plane figure.



Instructional Strategies: (3.MD.5-7)

Students can cover rectangular shapes with tiles and count the number of units (tiles) to begin developing the idea that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.

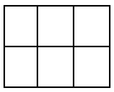
The area of a rectangle can be determined by having students lay out unit squares and count how many square units it takes to completely cover the rectangle completely without overlaps or gaps. Students need to develop the meaning for computing the area of a rectangle. A connection needs to be made between the number of squares it takes to cover the rectangle and the dimensions of the rectangle. Ask questions such as:

- What does the length of a rectangle describe about the squares covering it?
- What does the width of a rectangle describe about the squares covering it?

Continued next page

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Ask questions about the dimensions if students do not make these discoveries. For example:

- How do the squares covering a rectangle compare to an array?
- How is multiplication used to count the number of objects in an array?



Students should also make the connection of the area of a rectangle to the area model used to represent multiplication.

This connection justifies the formula for the area of a rectangle.

Provide students with the area of a rectangle (i.e., 42 square inches) and have them determine possible lengths and widths of the rectangle. Expect different lengths and widths such as, 6 inches by 7 inches or 3 inches by 14 inches.

<u>Common Misconceptions:</u>

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

Domain: **Measurement and Data (MD)**
Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
Standard: **3.MD.6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
Standards for Mathematical Practices to be emphasized: MP.2. Reason abstractly and quantitatively. MP.3. Model with mathematics MP.5. Use appropriate tools strategically. MP.6. Attend to precision.
Connections: See 3.MD.5
Explanations and Examples: Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. An interactive whiteboard may also be used to display and count the unit squares (area) of a figure. **See Instructional Strategies 3.MD.5**

Domain: **Measurement and Data (MD)**
Cluster: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard: **3.MD.7**. Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

Connections:

See 3.MD.5

Explanations and Examples:

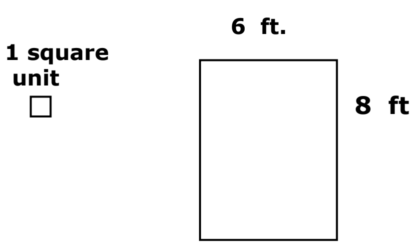
Students should tile rectangle then multiply the side lengths to show it is the same.

To find the area one could count the squares or multiply 3 x 4 = 12.

1	2	3	4
5	6	7	8
9	10	11	12

Students should solve real world and mathematical problems.

Example:
Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



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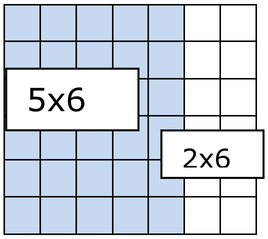
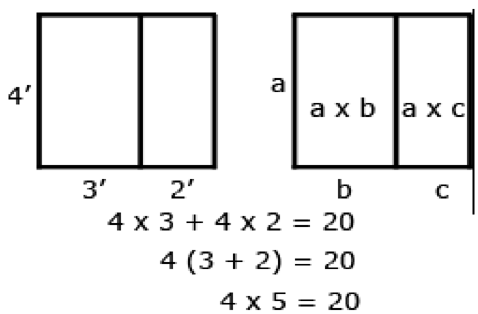
This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.



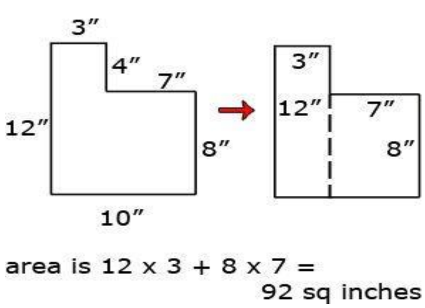
Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.

Example:
Joe and John made a poster that was 4ft. by 3ft. Melisa and Barb made a poster that was 4ft. by 2ft. They placed their posters on the wall side-by-side so that that there was no space between them. How much area will the two posters cover?

Students use pictures, words, and numbers to explain their understanding of the distributive property in this context.



Example:
Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



Domain: **Measurement and Data (MD)**

Cluster: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Standard: **3.MD.8.** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Standards for Mathematical Practices to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.

Connections:

This cluster is connected to the Third Grade Critical Area of Focus #3, **Developing understanding of the structure of rectangular arrays and of area**.
Measure and estimate lengths in standard units. Grade 2 MD 1 – 4
Relate addition and subtraction to length. Grade 2 MD 5 – 6

Explanations and Examples:

Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Continued next page

Instructional Strategies :

Students have created rectangles before when finding the area of rectangles and connecting them to using arrays in the multiplication of whole numbers. To explore finding the perimeter of a rectangle, have students use nonstretchy string. They should measure the string and create a rectangle before cutting it into four pieces. Then, have students use four pieces of the nonstretchy string to make a rectangle. Two pieces of the string should be of the same length and the other two pieces should have a different length that is the same. Students should be able to make the connection that perimeter is the total distance around the rectangle.

Geoboards can be used to find the perimeter of rectangles also. Provide students with different perimeters and have them create the rectangles on the geoboards. Have students share their rectangles with the class. Have discussions about how different rectangles can have the same perimeter with different side lengths.

Students experienced measuring lengths of inches and centimeters in Grade 2. They have also related addition to length and writing equations with a symbol for the unknown to represent a problem.

Once students know how to find the perimeter of a rectangle, they can find the perimeter of rectangular-shaped objects in their environment. They can use appropriate measuring tools to find lengths of rectangular-shaped objects in the classroom. Present problems situations involving perimeter, such as finding the amount of fencing needed to enclose a rectangular shaped park, or how much ribbon is needed to decorate the edges of a picture frame. Also present problem situations in which the perimeter and two or three of the side lengths are known, requiring students to find the unknown side length.

Students need to know when a problem situation requires them to know that the solution relates to the perimeter or the area. They should have experience with understanding area concepts when they recognize it as an attribute of plane figures. They also discovered that when plane figures are covered without gaps by n unit squares, the area of the figure is n square units.

Students need to explore how measurements are affected when one attribute to be measured is held constant and the other is changed. Using square tiles, students can discover that the area of rectangles may be the same, but the perimeter of the rectangles varies. Geoboards can also be used to explore this same concept.

Common Misconceptions:

Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter. Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.

Third Grade Mathematics
Extended Common Core State Standards Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.
North Carolina DOE

Third Grade Mathematics
Measurement and Data (3.MD)

Common Core State Standards		Essence	Extended Common Core	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		**Solve problems involving measurement**	**Solve problems with measurements involving time and length**	
Cluster	1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).6 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.		Cluster	1. Recall names of the months. 2. Use a full day schedule to order the events of the day. 3. Compare two objects using direct comparison of length. 4. Solve problems using appropriate vocabulary to describe differences in length (e.g. more, less, same). 5. Use standard customary unit to measure length (inch).
Represent and interpret data		**Represent and interpret data**	**Represent and interpret data**	
	3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.			6. Organize and represent data using a line plot. 7. Title and label axis of graph. 8. Answer questions posed about the collected data.

Domain: **Geometry (G)**

Cluster: **Reason with shapes and their attributes.**

Standard: **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Standards for Mathematical Practices to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

MP.7. Look for and make use of structure.

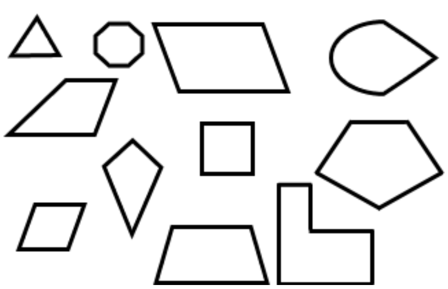
Connections: (3.G.1-2)

This cluster is connected to the Third Grade Critical Areas of Focus #3**, Developing understanding of the structure of rectangular arrays and of area** and #4, **Describing and analyzing two-dimensional shapes**.

Reason with shapes and their attributes. (Grade 2 G 3)

Explanations and Examples:

In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration).
Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures.
They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.



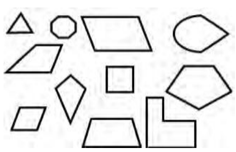
Students should classify shapes by attributes and drawing shapes that fit specific categories.
For example, parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.

Continued next page

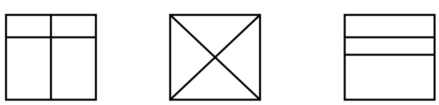
Example:
Draw a picture of a quadrilateral. Draw a picture of a rhombus.
How are they alike? How are they different?
Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

<u>Instructional Strategies</u> (3.G.1-2)
In earlier grades, students have experiences with informal reasoning about particular shapes through sorting and classifying using their geometric attributes. Students have built and drawn shapes given the number of faces, number of angles and number of sides.

The focus now is on identifying and describing properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification. Development in focusing on the identification and description of shapes' properties should include examples and nonexamples, as well as examples and nonexamples drawn by students of shapes in a particular category. For example, students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes.



In Grade 2, students partitioned rectangles into two, three or four equal shares, recognizing that the equal shares need not have the same shape. They described the shares using words such as, halves, thirds, half of, a third of, etc., and described the whole as two halves, three thirds or four fourths. In Grade 4, students will partition shapes into parts with equal areas (the spaces in the whole of the shape). These equal areas need to be expressed as unit fractions of the whole shape, i.e., describe each part of a shape partitioned into four parts as $1/4$ of the area of the shape.



Have students draw different shapes and see how many ways they can partition the shapes into parts with equal area.

<u>Common Misconceptions</u>: (3.G.1-2)
Students may identify a square as a "nonrectangle" or a "nonrhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Domain: **Geometry**
Cluster: **Reason with shapes and their attributes.**
Standard: **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Standards for Mathematical Practices to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.

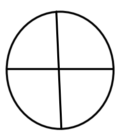
Connections:

See 3.G.1

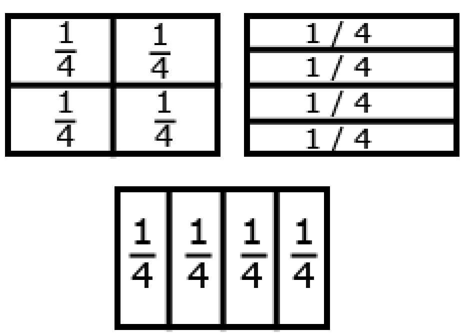
Explanations and Examples:

This standard builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths.

Example:
This figure was partitioned/divided into four equal parts. Each part is ¼ of the total area of the figure.



Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.



Third Grade Mathematics
Extended Common Core State Standards Mathematics

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North Carolina DOE

Third Grade Mathematics Geometry (3.G)				
Common Core State Standards		**Essence**	**Extended Common Core**	
Reason with shapes and their attributes		**Reason with shapes and their attributes**	**Reason with shapes and their attributes**	
Cluster	1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*		Cluster	1. Recognize the attributes of a rhombus and other quadrilaterals. 2. Partition shapes into equal halves. Express the area of each part as the fraction ½. Demonstrate understanding that this is 1 or 2 parts.

	EXTENDED 3.G				

TABLE 1. Common addition and subtraction situations.[34]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[35]
Put Together/ Take Apart[36]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[37]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[34] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
[35] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.
[36] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.
[37] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

TABLE 2. Common multiplication and division situations.[38]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 × 6 = ?	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[39] Area[40]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[38] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[39] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[40] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.